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WS 03/04

# Introduction to Quantum Computation
# Übung 1

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**1.1** Let $V$ and $W$ be vector spaces with bases $\{v_1, \dots, v_n\}$ and $\{w_1, \dots, w_m\}$, respectively. Prove that

$$\{v_j \otimes w_k \mid j = 1, \dots, n,\ k = 1, \dots, m\}$$

is a basis of $V \otimes W$.

**1.2** Show that the columns of a unitary matrix, when taken out and used as $n$ column vectors, form an orthonormal basis of $\mathbb{C}^n$. Show the same is true of the rows, when taken out and turned into column vectors.

**1.3** **a)** Construct an identification between $U(1)$, the group of $1 \times 1$ unitary matrices, and $S^1$, the unit circle in the complex plane $\mathbb{C}$.

**b)** Construct an identification between $SU(2)$, the group of $2 \times 2$ unitary matrices of determinant one, and $S^3$, the unit sphere in $\mathbb{R}^4$.

**1.4** Prove that the EPR states are actually entangled. That is, show that there do not exist any (single qubit) states $|\psi\rangle$ and $|\zeta\rangle$ such that $|\beta_{00}\rangle = |\psi\rangle \otimes |\zeta\rangle$, and likewise for $|\beta_{01}\rangle$, $|\beta_{10}\rangle$ and $|\beta_{11}\rangle$.

**1.5** Verify that the EPR states given in class can also be written as

$$|\beta_{ab}\rangle = \frac{|0b\rangle + (-1)^a |1\bar{b}\rangle}{\sqrt{2}}.$$

**1.6** Use the Hadamard and CNOT gates to define a 2-qubit gate which takes $|ab\rangle$ to the EPR state $|\beta_{ab}\rangle$ (an entanglement machine!).

**1.7** Let $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ be the Hadamard 1-qubit gate. Find a formula for $H^{\otimes n} : \mathbb{C}^{2n} \to \mathbb{C}^{2n}$, or at least for $H^{\otimes n}(|0\rangle \otimes \cdots \otimes |0\rangle)$.

*Hint:* Write $|b_1\rangle \otimes \cdots \otimes |b_n\rangle$, where each $b_j \in \{0, 1\}$, as $|b_1 \dots b_n\rangle = |x\rangle$, thinking of $x \in \{0, 1\}^n$ or even $x \in \{0, \dots, 2^n - 1\}$.

**1.8** What is the action of a CNOT on the control bit when the target bit is set to $|0\rangle - |1\rangle$? Now see what happens when you conjugate CNOT by $H^{\otimes 2}$, *i.e.*, you apply the Hadamard gate to both of CNOT's inputs and both of its outputs.