

Introduction to Quantum Computation

Übung 6

12.12.2002

- 6.1** Give an explicit example of a quantum operation coming from a unitary transformation on the total system/environment state space which does *not* act as a unitary transformation would on the density matrix of the system alone – be very concrete here, with choices of Hilbert spaces and matrices.
- 6.2** Show that a pure state of a composite quantum system is entangled if and only if its reduction to a subsystem is mixed.
- 6.3** ([NC], Exercise 8.4): Suppose we have a single-qubit principal system interacting with a single-qubit environment through the transform

$$U = P_0 \otimes I + P_1 \otimes X,$$

where X is the usual (Pauli) matrix and P_0 and P_1 are the projections on the computational coordinate axes (*e.g.*, $P_0 = |0\rangle\langle 0|$), and the first factor acts on the principal system. Give the quantum operation for this process, in the operator-sum representation, assuming the environment starts in the state $|0\rangle$. What does this have to do with measurement?

- 6.4** Describe in terms of quantum operations the quantum coin-tossing you invented to solve problem 2.8 from Übung 2.
- 6.5** Explain how to get from equation (8.13) to (8.14) in [NC].